ピックの定理の拡張

Pick's theorem computes the area of a polygon on the square lattice by the interior and boundary points. We came up with a new formula similar to this which is applicable to equilateral triangle and hexagonal lattices. In the case of equilateral triangle, we found the equation using mathematical induction. In the case of hexagonal lattices, we worked out a formula using the new idea of boundary characteristic (the edges that extends locally into the exterior of the polygon – the edges that extends locally into the interior of the polygon) and it proved to be accurate. The idea is also true for square and equilateral triangle. In this way, we were able to expand Pick's theorem.

キーワード ,

grid point, equilateral triangle lattice, hexagonal lattice, basic form, Pick's theorem

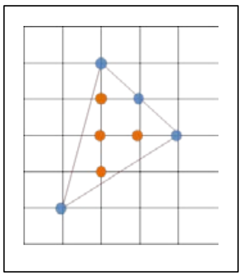
序論

本論

S: x: y:

$$S = x + \frac{1}{2} y - 1$$

$$S = 4 + \frac{1}{2} \times 4 - 1 = 5$$

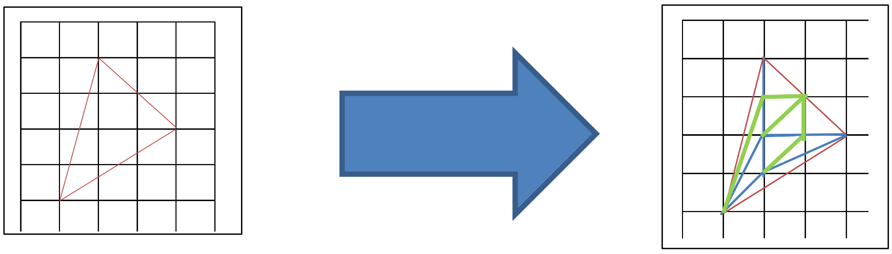


1.

2.

3.

$\frac{1}{2}$



=1

$x = 0$, $y = 3$ $S = 0 + \frac{3}{2} - 1 = \frac{1}{2}$

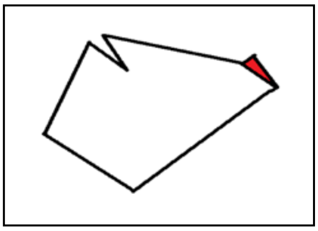
n= k

$\frac{1}{2}$ k $S = x + \frac{1}{2}y - 1 = \frac{1}{2}k$

$S = \frac{1}{2}(k + 1)$

n= $k + 1$

x $y + 1$

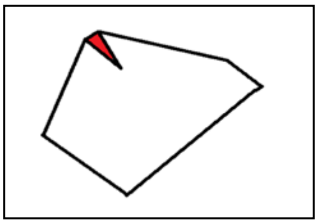


$S = x + \frac{1}{2}(y + 1) - 1$=$(x + \frac{1}{2}y - 1)$+$\frac{1}{2}$

=$\frac{1}{2}k + \frac{1}{2}$

=$\frac{1}{2}(k + 1)$

1 1 x+1, y −1



$S = (x + 1) + \frac{y-1}{2} - 1$

=$(x + \frac{1}{2}y - 1)$+$\frac{1}{2}$

=$\frac{1}{2}k + \frac{1}{2}$

=$\frac{1}{2}(k + 1)$

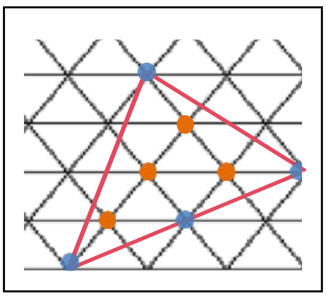
n= k+1

n

S: x: y:

$ax + by + c = S$

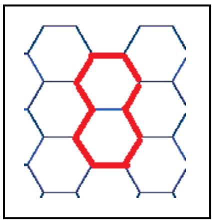
$S = 2\left(x + \frac{1}{2}y - 1\right)$

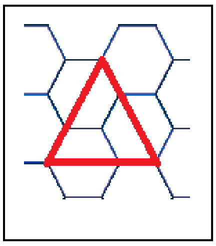


S:　x:　y:

$S=\frac{1}{2}\left(x+\frac{1}{2}y-1\right)$

$\frac{1}{2}$





$\frac{1}{2}$

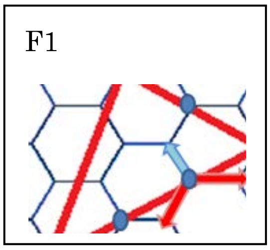
$$S=\frac{1}{2}\left(x+\frac{1}{2}y-1\right)$$

z

z

z :(　)　(　)

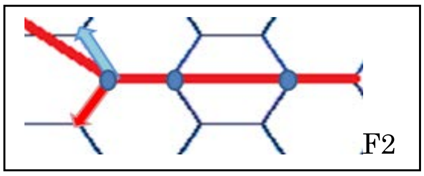
F1　F2

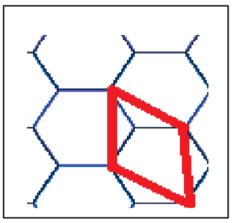


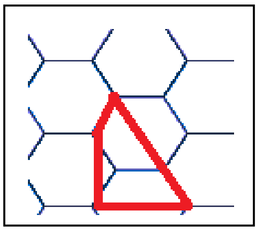
F1　2

1

F2　1　1







$S=\frac{1}{12}(6x+3y+z-12)$

$$S=\frac{1}{12}(6x+3y+z-12)$$

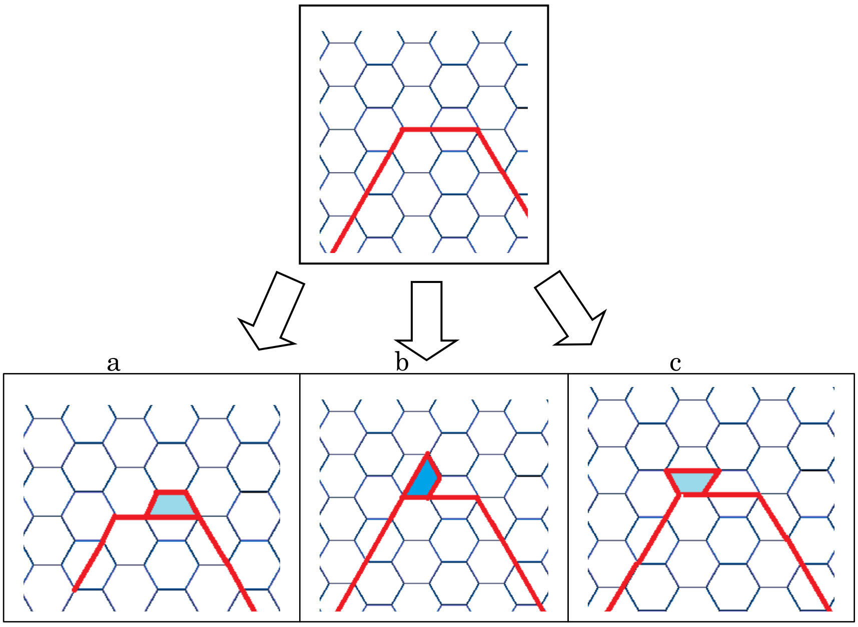
$\frac{1}{2}$

( ) =1 $S=\frac{1}{12}(6\times 0+3\times 4+z-12)$ $\frac{1}{2}$

( ) =2 $S=\frac{1}{12}(6\times 0+3\times 4+z-12)$ $\frac{1}{2}$

S $\frac{1}{2}$( )

( ) 3



a $x_a=x+0$ $y_a=y+2$ $z_a=z+0$

S $\frac{1}{12}(6(\quad+0)+\quad(\quad+2)+(z+0)-12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

b $x_b=x$ $y_b=y+2$ $z_b=z+0$

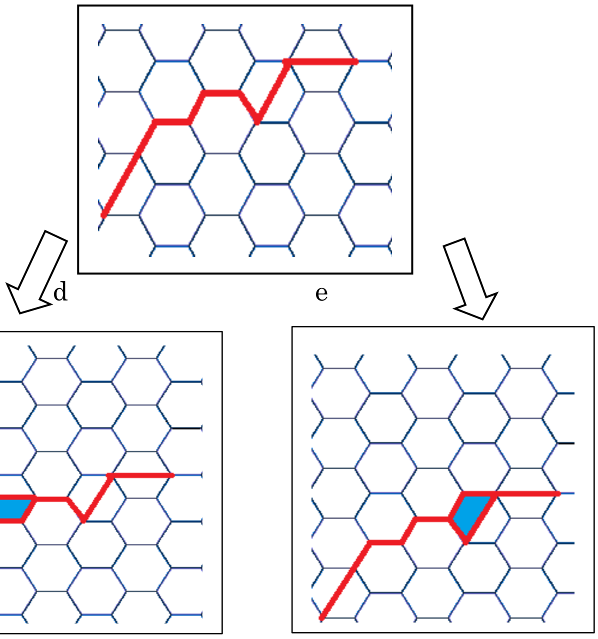
S $\frac{1}{12}(6(\quad+0)+3(\quad+2)+(z+0)-12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

c $x_c=x+0$ $y_c=y+2$ $z_c=z+0$

S $\frac{1}{12}(6(x+0)+3(y+2)+(z+0)-12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

a b c

( )



d $x_d = x{+}1$ $y_d = y + 0$ $z_d = z{+}0$

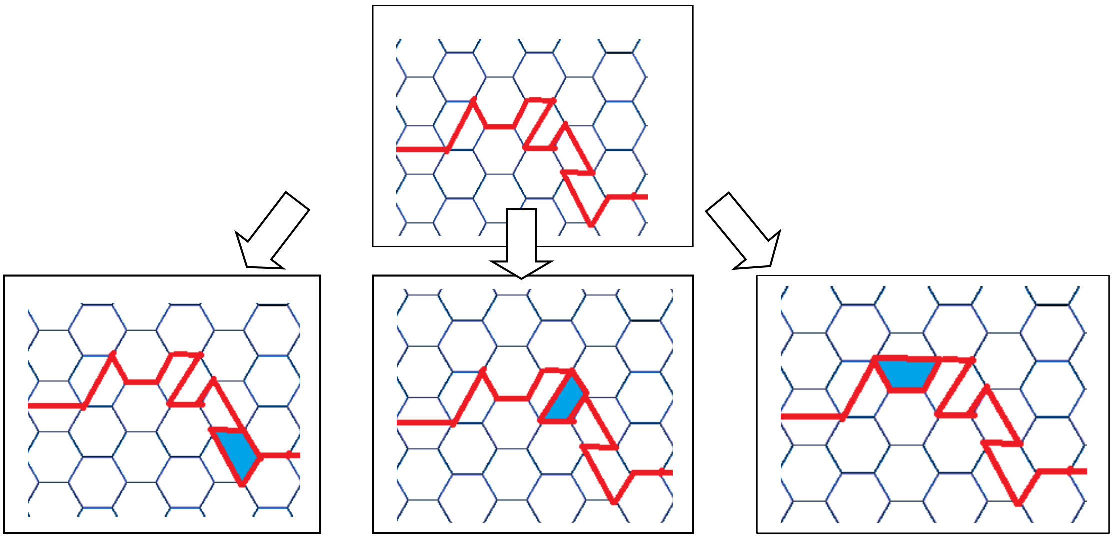
S $\frac{1}{12}(6(x+1) + 3(y+0) + (z+0) - 12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

e $x_e = x$ $y_e = y$ $z_e = z$

S $\frac{1}{12}(6(x+1) + 3(y+0) + (z+0) - 12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

d e

( )



f g h

f x $= x{+}2$ y $= y - 2$ z $= z{+}0$

S $\frac{1}{12}(6(x+2) + 3(y-2) + (z+0) - 12)$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}(k+1)$

g $x = x \quad 2 \quad y = y-2 \quad z = z+0$

S $\frac{1}{12}(6(x+2)+3(y-2)+(z+0)-12) \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}(k+1)$

h $x = x+2 \quad y = y-2 \quad z = z+0$

S $\frac{1}{12}(6(x+2)+3(y-2)+(z+0)-12) \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}(k+1)$

( ) ( ) ( ) $S = \frac{1}{12}(6x+3y+z-12)$

z

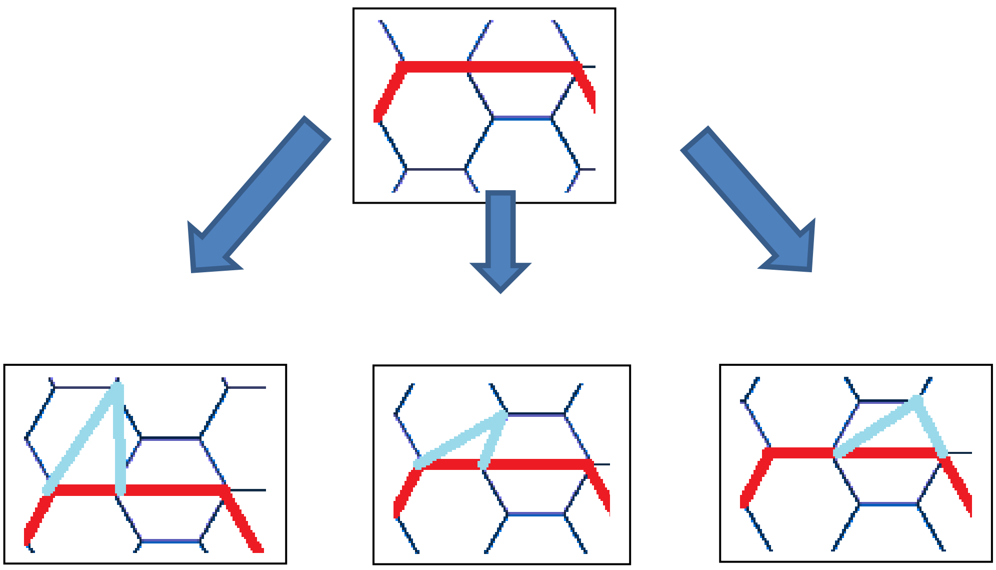
$S = \frac{1}{2}\left(x+\frac{1}{2}y-1\right)$

$\frac{1}{2}$

$x \quad y \quad z$

$S = \frac{1}{12}(6x+3y+z-12)$

3



A B

$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x \quad y_b = y+1 \quad z_b = z+1$

$$S = \frac{1}{12}\left(6x \quad +3y \quad +z \quad -12\right) = \frac{1}{12}\{6x+3(y+1)+(z+1)-12\} = \frac{1}{12}(6x+3y+z-12)+\frac{1}{3}$$

$$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x \quad y_b = y + 1 \quad z_b = z - 1$$

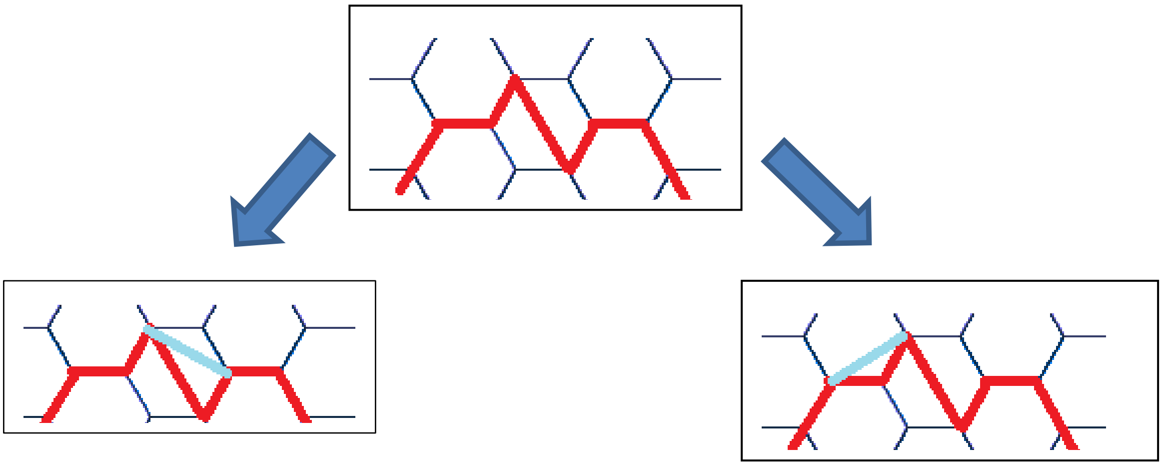
$$S = \frac{1}{12}\left(6x \quad + 3y \quad + z \quad - 12\right) = S = \frac{1}{12}\{6x + 3(y + 1) + (z - 1) - 12\} = \frac{1}{12}(6x + 3y + z - 12) + \frac{1}{6}$$

$$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x \quad y_b = y + 1 \quad z_b = z + 1$$

$$S = \frac{1}{12}\left(6x \quad + 3y \quad + z \quad - 12\right) = S = \frac{1}{12}\{6x + 3(y + 1) + (z + 1) - 12\} = \frac{1}{12}(6x + 3y + z - 12) + \frac{1}{3}$$

$x \quad y \quad z$

$S = \frac{1}{12}(6x + 3y + z - 12)$



A　　B

$$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x + 1 \quad y_b = y - 1 \quad z_b = z + 1$$

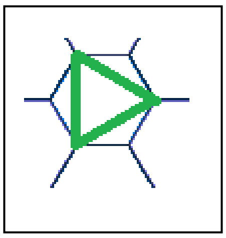
$$S = \frac{1}{12}\left(6x \quad + 3y \quad + z \quad - 12\right) = \frac{1}{12}\{6(x + 1) + 3(y - 1) + (z + 1) - 12\}$$

$$= \frac{1}{12}(6x + 3y + z - 12) + \frac{1}{3}$$

$$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x + 1 \quad y_b = y - 1 \quad z_b = z - 1$$

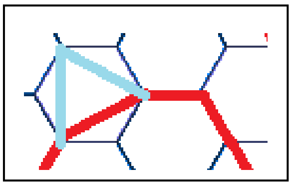
$$S = \frac{1}{12}\left(6x \quad + 3y \quad + z \quad - 12\right) = S = \frac{1}{12}\{6(x + 1) + 3(y - 1) + (z - 1) - 12\}$$

$$= \frac{1}{12}(6x + 3y + z - 12) + \frac{1}{3}$$



$x = 0 \quad y = 3 \quad z = 9$

$$S = \frac{1}{12}(6x + 3y + z - 12) = \frac{1}{12}(0 + 9 + 9 - 12) = \frac{1}{2}$$



A B

$x_a = x \quad y_a = y \quad z_a = z \qquad x_b = x \quad y_b = y + 1 \quad z_b = z + 3$

$$S = \frac{1}{12}\left(6x \quad + 3y \quad + z \quad - 12\right) = \frac{1}{12}\{6x + 3(y + 1) + (z + 3) - 12\}$$

$$= \frac{1}{12}(6x + 3y + z - 12) + \frac{1}{2}$$

$\frac{1}{12}(6x + 3y + z - 12)$

z

今後の予定

謝辞

TA

参考文献

P.33 P.46

THE HOENYCOMB CONJECTURE Thomas C,HALES

http://math.artet.net/?eid=1161087

http://trypophobia.net/trypophobia/honeycomb-trypo...